In this survey talk we aim to clarify the close relationships between the operators of the generalized fractional calculus (GFC), some classes of generalized hypergeometric functions and generalizations of the classical integral transforms. The GFC developed in [1] is based on the essential use of the Special Functions (SF). The generalized (multiple) fractional integrals and derivatives are defined by single (differ) integrals with Meijer’s G- and Fox H-functions in the kernels, but represent also products of classical Erdelyi-Kober operators. This theory is widely illustrated by numerous special cases and applications to different areas of Applied Analysis: SF, Integral Transforms (IT), operational calculus, differential and integral equations, hyper-Bessel and Gelfond-Leontiev operators, geometric function theory, etc. Here we focus our attention to the first two topics of applications, SF and IT.

First of all, the GFC operators are defined by means of IT whose kernels are SF. Next, we provide a new sight on the SF (the generalized hypergeometric functions \( \genfrac{}{}{0pt}{}{p}{\nu} \) and \( \genfrac{}{}{0pt}{}{p}{\Psi} \)) as operators of GFC of 3 simplest elementary functions, depending on either \( p<q, \ p=q, \ p=q+1 \) ([1],[2],[7]). On this GFC base, new classification of the SF is proposed, along with new integral and differential representations (generalizations of Poisson and Euler integrals and Rodrigues formulas). Besides, the GFC operators are interpreted as Gelfond-Leontiev operators generated by the multi-index Mittag-Leffler (M-L) functions. Basic properties of this new class of SF of FC are studied and their applications in fractional order mathematical models of are emphasized, [4],[6].

By means of GFC operators in the role of transmutation operators, we introduce and study important generalizations of the Laplace integral transform. The Borel-Dzrbashjan, Meijer, Kratzel, Obrechkoff, multi-index Borel-Dzrbashjan and other useful integral transforms are shown to be special cases. FC and SF techniques allow to develop the theory of these generalized IT, including operational rules, convolutions, tables of images, real and complex inversion formulas, Abelian type theorems, etc; [1],[3],[5]. Vice versa, we need to emphasize the role of the Obrechkoff integral transform and related hyper-Bessel operators [3],[1], as giving initial hint to develop the GFC [1].